No calc.

3. (10 points) Without using a calculator, calculate the following. If the expression does not exist or is undefined, say so and explain why. Make sure to show your work, if there is any work to be shown! If it would help you, you can fill out the unit circle on the next page.

✓ (a) $\cos(60°) = \frac{1}{2}$

✓ (b) $\sin\left(\frac{7\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

(c) $\tan\left(-\frac{2\pi}{3}\right) = \frac{\sin(-\frac{2\pi}{3})}{\cos(-\frac{2\pi}{3})} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{-1} = \boxed{\sqrt{3}}$

1.5 (d) $\csc(-540°) = \frac{1}{\sin(-540)} = \frac{1}{0} \Rightarrow \boxed{\text{undefined}}$

✓✓ $2\pi = \frac{12\pi}{6}$ (e) $\cot\left(\frac{41\pi}{6}\right) = \cot\left(\frac{29\pi}{6}\right) = \cot\left(\frac{17\pi}{6}\right) = \cot\left(\frac{5\pi}{6}\right) = \frac{\cos}{\sin} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \boxed{-\sqrt{3}}$

1.5 (f) $\sin^{-1}\left(\frac{2}{\sqrt{2}}\right) = \underline{\text{undefined}}$ b/c $\frac{2}{\sqrt{2}} = \frac{\sqrt{2}\cdot\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} = \sqrt{2} \approx 1.4 > 1$

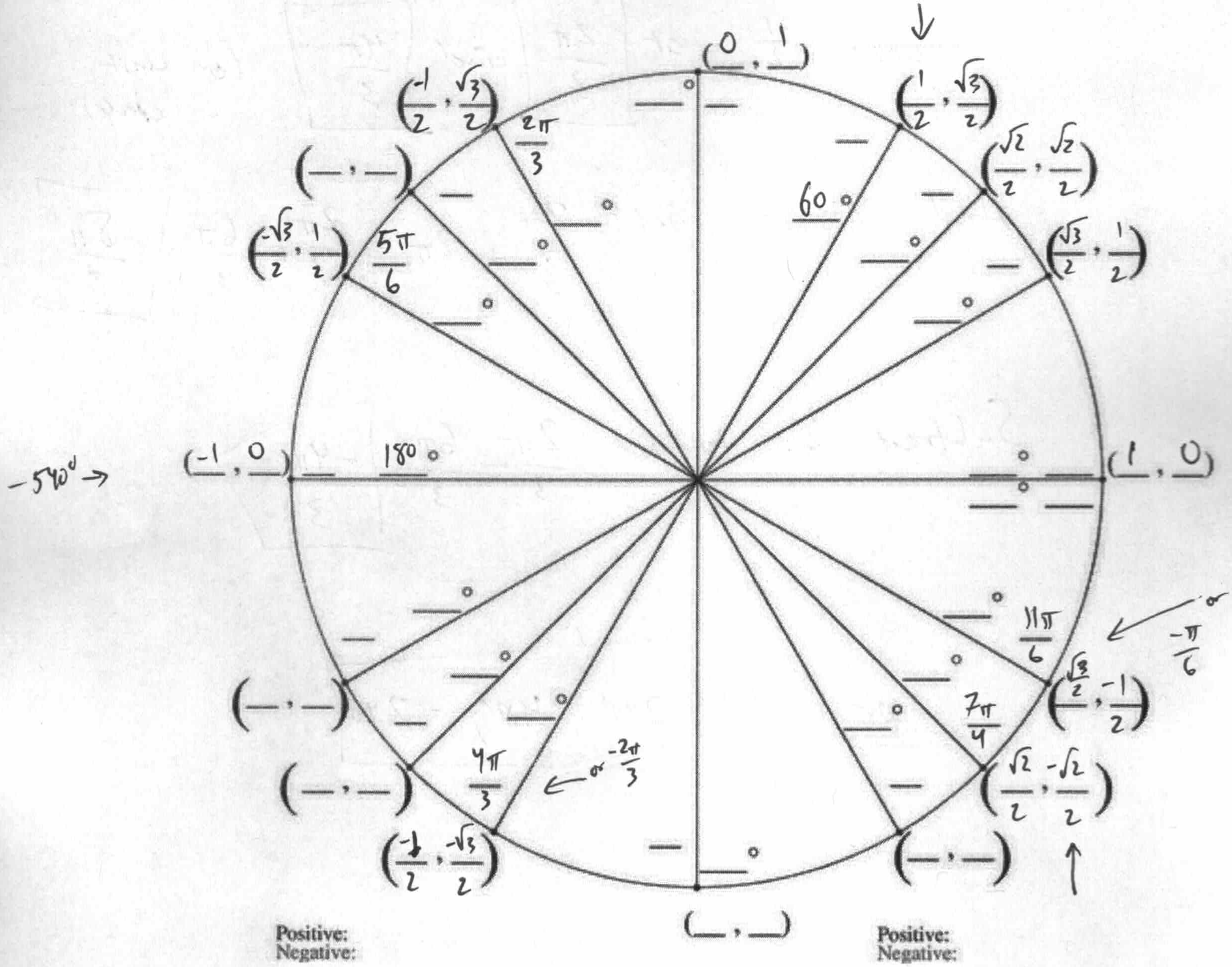
and $\sin^{-1}$ is only defined for $[-1, 1]$

✓✓ (g) $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

Need $\sin = \pm\frac{1}{2}$ $\cos = \pm\frac{\sqrt{3}}{2}$, and one of them negative $\Rightarrow$ 4th quadrant

$\Rightarrow \boxed{-\frac{\pi}{6}}$

Unit Circle for you to fill in, if you would like to:



No calc

4. (3 points) For the equation $\cos\theta = \frac{-1}{2}$, find 4 different solutions in radians or degrees: one negative solution, two solutions between 0 and 2π, and one solution greater than 2π.

$\cos\theta = \frac{-1}{2}$ at $\boxed{\frac{2\pi}{3}}$ and $\boxed{\frac{4\pi}{3}}$ (on unit circle)

Add 2π to get $\frac{2\pi}{3} + 2\pi = \frac{2\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{8\pi}{3}}$

Subtract 2π to get $\frac{2\pi}{3} - \frac{6\pi}{3} = \boxed{\frac{-4\pi}{3}}$

or $\boxed{120^\circ, 240^\circ, 480^\circ, -240^\circ}$

✓ for 1/4 or all wrong but work shown

✓✓ for 2/4 and some work

2.5 for 3/4 "

3

1. ($\cancel{5}$ 3 points) Solve the equation

$$\log_3(x+8) + \log_3(x) = 2$$

either algebraically or graphically.

Whichever method you choose, you should explain/show your work.

$$\log_3\left[(x+8)(x)\right] = 2 \qquad 0.5$$

$$x(x+8) = 3^2 \qquad 0.5$$

$$x^2+8x-9 = 0 \qquad 0.5$$

$$(x+9)(x-1) = 0 \qquad 0.5$$

$$x = -9 \quad \text{or} \quad x = 1 \qquad 0.5$$

check $x = -9 \Rightarrow \log_3(-9+8) + \log_3(-9)$

is <u>no</u> good

both are undefined

b/c cant do log(neg)

0.5

$$\boxed{x = 1}$$

No calc

5. (5 points) (a) Starting with the identity $1 + \cot^2\theta = \csc^2\theta$, multiply both sides of the equation by $\sin^2\theta$ to find the trigonometric identity relating the values of $\sin\theta$ and $\cos\theta$. Show your work.

$$1 + \cot^2\theta = \csc^2\theta$$

✓ $$\sin^2\theta\left(1 + \frac{\cos^2\theta}{\sin^2\theta}\right) = \left(\frac{1}{\cancel{\sin^2\theta}}\right)\cancel{\sin^2\theta}$$

✓ $$\sin^2\theta + \cancel{\sin^2\theta}\left(\frac{\cos^2\theta}{\cancel{\sin^2\theta}}\right) = 1$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

(b) If $\sin\theta = \frac{-1}{4}$ and θ is in the fourth quadrant, use trigonometric identities to find the values of (i) $\cos\theta$, (ii) $\sec\theta$, and (iii) $\cot\theta$.

⇒ exact

(i) $\left(-\frac{1}{4}\right)^2 + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \frac{1}{16} \Rightarrow \cos\theta = \pm\sqrt{\frac{15}{16}}$

✓

4^{th} Quad ⇒ cos is positive ~~negative~~

so $\boxed{\cos\theta = \frac{\sqrt{15}}{4}}$

✓ (ii) $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{\sqrt{15}}{4}\right)} = \boxed{\frac{4}{\sqrt{15}}}$

(iii) $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\left(\frac{\sqrt{15}}{4}\right)}{\left(-\frac{1}{4}\right)} = \left(\frac{\sqrt{15}}{\cancel{4}}\right)\cdot\left(\frac{-\cancel{4}}{1}\right) = \boxed{-\sqrt{15}}$

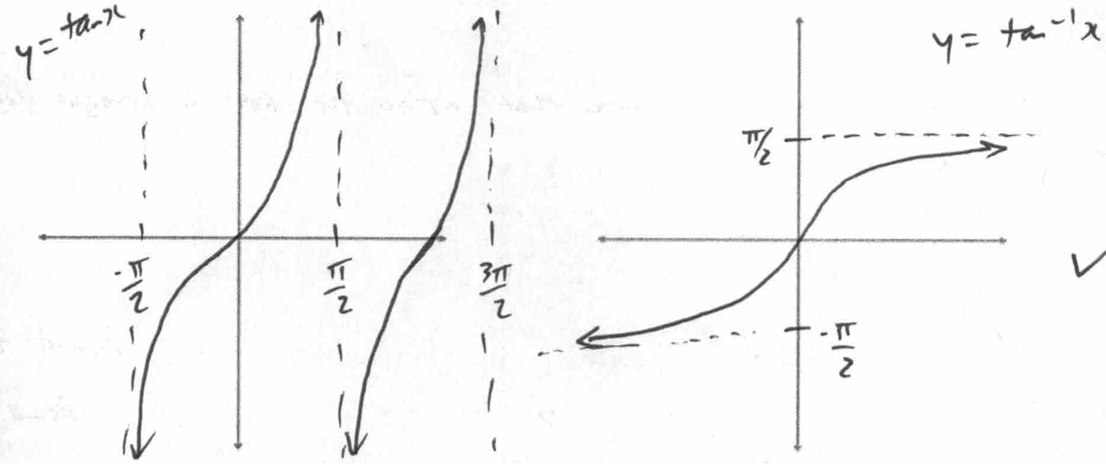
✓

No calc.

8. (5 points) The graph of the arctangent function ($y = \arctan x$ or $y = \tan^{-1} x$) is unusual because it has two different horizontal asymptotes.

(a) *State* what the two different horizontal asymptotes are (one for infinity and the other for negative infinity) and

(b) *Explain* why those are the horizontal asymptotes. You will likely need to sketch the graph of $y = \tan x$, and discuss inverse functions, in order to give a good explanation.



✓

✓✓ (a) $y = \frac{\pi}{2}$ and $y = \frac{-\pi}{2}$

(b) $\tan^{-1} x$ is the inverse of $\tan x$, defined for

✓ $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. As $x \to \frac{\pi}{2}$, $\tan x \to \infty$ b/c

$\tan x = \frac{\sin x}{\cos x}$, and as $x \to \frac{\pi}{2}$, $\cos x \to 0$ and $\sin x \to 1$,

✓ and dividing by zero makes $\tan x$ get very (∞) large.

so as $x \to \frac{\pi}{2}$, $y = \tan x \Rightarrow \infty$. Thus for the inverse,

as $x \to \infty$, $y = \tan^{-1} x \Rightarrow \frac{\pi}{2}$

Need calc

2. (5 points) (a) Suppose the element Mongoosium has a half-life m. Starting with the general form of the exponential growth/decay model $A = Pe^{rt}$, prove that $r = \dfrac{\ln(1/2)}{m}$ (or, equivalently, $r = -\dfrac{\ln 2}{m}$). (Hint: it may help to consider a specific example, say you have 100 grams of Mongoosium, and then do the necessary calculations from there.)

After m years, $A = \frac{1}{2}P$. so

$$\frac{\frac{1}{2}P}{P} = \frac{Pe^{rm}}{P}$$

$$\frac{1}{2} = e^{rm}$$

$$\ln\left(\tfrac{1}{2}\right) = \ln\left(e^{rm}\right)$$

$$\frac{\ln\left(\frac{1}{2}\right)}{m} = \frac{rm}{m}$$

$$\boxed{r = \frac{\ln(1/2)}{m}}$$

(b) Suppose the half-life of Mongoosium is $m = 1732$ years. A staff made of Mongoosium is unearthed at a geological site. The ~~panda~~ staff has only 6% of the Mongoosium left of the original amount. How long ago was the staff buried? Round to the nearest year.

$A = 0.06P$, need to solve for t. First find

$r = \dfrac{\ln(1/2)}{1732}$ and then

$$0.06\cancel{P} = \cancel{P}e^{\left(\frac{\ln(1/2)}{1732}\right)t}$$

$$\ln(0.06) = \left(\frac{\ln(1/2)}{1732}\right)t$$

$$\frac{1732}{\ln(1/2)}\left(\ln(0.06)\right) = t$$

$$7030 = t$$

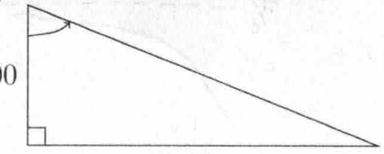
$\boxed{7030 \text{ years ago}}$

7. (5 points) Griffin is standing on the roof of the Smith tower (1000 feet high), and they attach a laser pointer to their fidget spinner, and spin it slowly, counterclockwise so that the laser pointer makes a full rotation every 4 minutes. At time t, the distance d along the ground (measuring the distance from the tower to the laser dot on the ground, as shown in the diagram below) is given by

$$d(t) = 1000 \tan\left(\frac{\pi t}{2}\right)$$

where d is measured in feet and t in minutes. Note: your calculator must be in *radians* mode for this function. Also note that at $t=0$, the light is straight down and $d=0$.

Griffin



(a) Find the distance d at the given times: (i) $t = 0.1$ and (ii) $t = 0.7$. Round to nearest whole #

(b) How long does it take for d to reach 3000 feet? Explain/show your work. Round to nearest 0.00

(c) What happens to d as t gets closer to 1 minute? Explain why this makes sense *both* by looking at the function *and* by what would happen in the diagram (real life).

1.5

(a) (i) $d(0.1) = 1000 \tan\left(\frac{\pi(0.1)}{2}\right) = 158.38 = \boxed{158 \text{ feet}}$

(ii) $d(0.7) = 1000 \tan\left(\frac{\pi(0.7)}{2}\right) = 1962.6 = \boxed{1963 \text{ feet}}$

2

(b) $\frac{3000}{1000} = \frac{1000 \tan\left(\frac{\pi t}{2}\right)}{1000}$

$\tan^{-1}(3) = \frac{\pi t}{2}$

$\frac{2}{\pi}\left(\tan^{-1}(3)\right) = t$

$\boxed{t = 0.795 \text{ minutes}}$

0.5

(c) as t gets close to 1 minute you get close to $\tan\left(\frac{\pi}{2}\right) = \infty$ is undefined! (d gets infinitely large) This makes sense, because The light will be pointing out horizontally and not touching the ground anymore at $t=1$, because it has gone a quarter spin around

With Calculator Portion Name: Key

- Note that once you get your calculator and turn in the No Calculator portion, you CANNOT return to that part of the test!
- If you need it, the law of cosines is $c^2 = a^2 + b^2 - 2ab\cos(C)$.
- If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

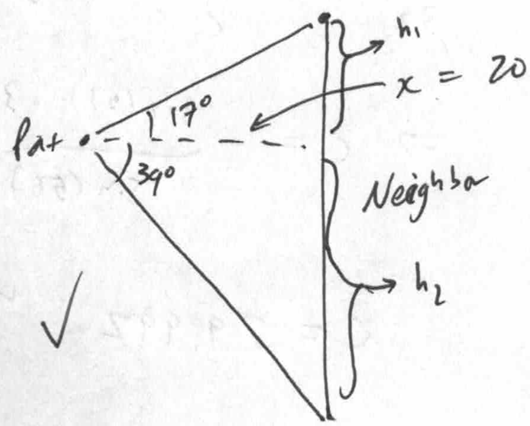
9. ($\cancel{8}$ 10 points) Pat is ~~looking out a window of their house~~ sitting in a tree, looking, straight across to Pat's neighbor's house, which is x feet away (in horizontal distance) from Pat~~'s house and h feet tall~~. The top of the neighbor's house is higher than Pat, and Pat measures the angle of elevation to the top of the neighbor's house to be 17°. The ground is below Pat, and Pat measures the angle of depression to the bottom of the neighbor's house to be 39°.

Note need degrees

(a) Draw a diagram of the situation.

(b) If the distance x between the houses is 20 feet, calculate the height h of the neighbor's house. Round to nearest 0.01 feet.

(a)



(b)

$$\tan(17) = \frac{h_1}{20} = \frac{opp}{adj}$$

$$20 \cdot \tan(17) = h_1$$

$$6.1146 = h_1$$

$$\tan(39) = \frac{h_2}{20}$$

$$h_2 = 20 \cdot \tan(39) = 16.1956$$

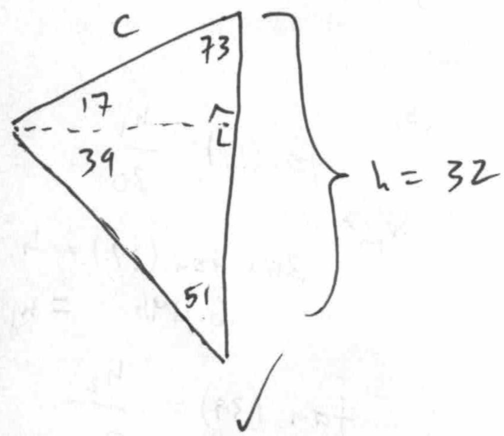
$$\boxed{\text{height} = h_1 + h_2 = 22.31 \text{ feet}}$$

Continuation of problem 9:

Pat is looking out a window of their house, straight across to Pat's neighbor's house, which is x feet away (in horizontal distance) from Pat's house and h feet tall. The top of the neighbor's house is higher than Pat, and Pat measures the angle of elevation to the top of the neighbor's house to be 17°. The ground is below Pat, and Pat measures the angle of depression to the bottom of the neighbor's house to be 39°.

Draw a diagram of the situation (should look same as in previous problem, part (a), but you *cannot* assume that x is still equal to 20).

$\Rightarrow$ (c) Now suppose you do not know the distance x between the houses, but you *do* know that the house is 32 feet tall. Can she calculate the distance ~~x~~? If not, explain why not. If so, calculate it (round to nearest 0.01) and explain how you got your answer.

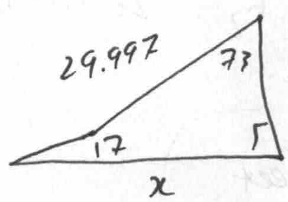


Law of sines says

$$\frac{\sin(56)}{32} = \frac{\sin(51)}{c}$$

$$\Rightarrow c = \frac{\sin(51) \cdot 32}{\sin(56)}$$

$$c = 29.997$$



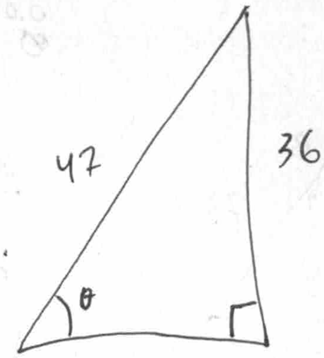
$$\cos(17) = \frac{x}{29.997}$$

$$\cos(17) \cdot 29.997 = x$$

$$\boxed{28.69 = x}$$

39 min.

10. (5 points) A 47 foot long ladder leans to touch the top of a building that is 36 feet tall. What is the angle of elevation (in degrees) of the ladder? In other words, what is the angle between the ladder and the ground? Round your answer to the nearest 0.01.



$$\sin\theta = \frac{36}{47}$$

$$\theta = \sin^{-1}\left(\frac{36}{47}\right)$$

$$\boxed{\theta = 49.99^\circ}$$

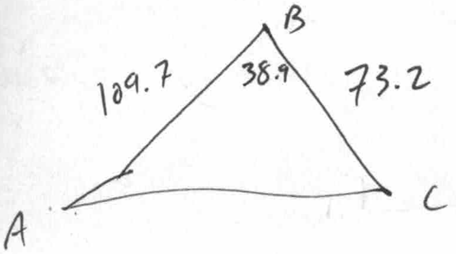
11. (5 points) For each of the given triangles ABC, do the following:

(a) determine if there is one, multiple, or no ways to solve the triangle.

(b) If there is one way to solve the triangle, find the length of the third side. If there are *multiple* ways to solve the triangle, find ~~the length of the third side~~ all the angles in the triangle that has an *obtuse* angle.

Round answers to the nearest 0.01, and explain your reasoning/show your work.

(i) $\angle B = 38.9°$, $c = 109.7$, and $a = 73.2$



One such Δ. Use Law of cosines

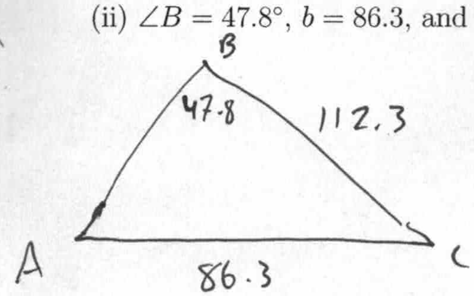
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

$$b^2 = (73.2)^2 + (109.7)^2 - 2(73.2)(109.7)\cos(38.9)$$

$$b^2 = 4893.68$$

$$b = \sqrt{4893.68} = 69.954 = \boxed{69.95}$$

(ii) $\angle B = 47.8°$, $b = 86.3$, and $a = 112.3$



Law of sines

$$\frac{\sin(47.8)}{86.3} = \frac{\sin(A)}{112.3}$$

$$A = \sin^{-1}\left(\frac{112.3(\sin(47.8))}{86.3}\right)$$

$$A = 74.57° \quad \text{or} \quad (180 - 74.57°) = 105.43$$

Since $105.43 + 47.8 < 180$, there are 2 possible Δs. The obtuse angled option has angles $\boxed{47.8,\ 105.43,}$ and

$$180 - 47.8 - 105.43 = \boxed{26.77°}$$